So we have this matrix,

$$G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and we multiply it by the row 2 of Pascal's triangle without its last element, alternating negation signs

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

and append it to G_1 to make

$$G_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

Similarly,

$$G_2 \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 6 \end{bmatrix} \text{ gives } G_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & -6 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$$G_3 \begin{bmatrix} -1 \\ 4 \\ -6 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 \\ 14 \\ -36 \\ 24 \end{bmatrix} \text{ gives } G_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 2 & -6 & 14 \\ 0 & 0 & 0 & 6 & -36 \\ 0 & 0 & 0 & 0 & 24 \end{bmatrix}$$

Of course, we could have started this one step sooner,

$$G_0 [1] = [1] [1] = [1] \text{ gives } G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So we know how to create the matrix, what does it do? Well, say we have a polynomial of degree 2

$$f(x) = 7 - 2x + x^2$$

Some values are

x	0	1	2	3
$f(x)$	7	6	7	10

It can be represented as a vector

$$\vec{f} = (7, -2, 1)$$

and can be transformed into $\vec{g}_0$ by premultiplying it by G_2

$$\vec{g}_0 = G_2\vec{f} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 2 \end{bmatrix}$$

If we add up the elements in $\vec{g}_0$ consecutively to get $\vec{g}_1$.

$$\vec{g}_1 = \begin{bmatrix} 7-3+2 \\ -3+2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 2 \end{bmatrix}$$

and again for $\vec{g}_2$ and $\vec{g}_3$

$$\vec{g}_2 = \begin{bmatrix} 6-1+2 \\ -1+2 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

$$\vec{g}_3 = \begin{bmatrix} 7+1+2 \\ 1+2 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 2 \end{bmatrix}$$

We see the first elements of $\vec{g}_0$, $\vec{g}_1$, $\vec{g}_2$, and $\vec{g}_3$ are 7, 6, 7, and 10 which correspond exactly with the values of $f(x)$ at $x = 0, 1, 2, 3$. This is always the case.