We transform the polynomial $f(x)$ of order n into $\vec{g}_0$ by

$$\vec{g}_0 = G_n \vec{f}$$

We can express movement of $\vec{g}$ over the values of x by this step matrix

$$S = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & 1 & 1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$s_{i,j} = \begin{cases} 0 & \text{if } i > j \\ 1 & \text{if } i \leq j \end{cases}$$

so that

$$\vec{g}_k = S^k \vec{g}_0$$

and remember,

$$f(k) = \vec{g}_{k,1} \quad \forall k \in \mathbb{Z}$$

We can also scale $f(x)$ by replacing every x with cx with c being the scale factor. This replacement is equivalent to premultiplying $\vec{f}$ by a matrix of the form

$$P(c) = \begin{bmatrix} c^0 & 0 & \cdots & 0 \\ 0 & c^1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c^n \end{bmatrix}$$

$$p_{i,j} = \begin{cases} p^{i-1} & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

So generally, if we want to get $\vec{g}_0$ to be a scaled, offset, and transformed from $f(x)$, we can use

$$\vec{g}_0 = S^k G P(c) \vec{f}$$

That involves raising S to an arbitrary power, however we intuitively know that stepping k times is equivalent to scaling the step size by k and stepping once. This gives us the useful relation

$$S^k = SGP\left(k\right)G^{-1}$$

So $\vec{g}_0$ can be computed better as

$$\vec{g}_0 = GP\left(\frac{c}{k}\right)G^{-1}SGP\left(k\right)\vec{f}$$