Polynomial Expansions

$$\left(\sum_{i=1}^{n} x_i\right)^r = (x_1 + x_2 + \cdots + x_n)^r = \sum_{k=1}^{A}\left(c_k \prod_{i=1}^{n} x_i^{r_{k,i}}\right)$$

x_i is a variable.

n is the number of different variables.

r is the original expression's exponent.

N is the number of terms in the final expansion.

$r_{k,i}$ is a the exponent of the k_{th} term's i_{th} variable in the final expansion.

$$\forall k \left(\sum_{i=1}^{n} r_{k,i} = r\right)$$

In each term, the exponents of the variables add up to the original exponent.

$$\sum_{k=1}^{A} c_k = n^r$$

The sum of the expansion's coefficients is equal to n^r.

$$c_k = \frac{n!}{\prod_{i=1}^{n} r_{k,i}!}$$

The coefficient of any given term is $n!$ divided by the product of the factorials of the variables' exponents in that term.

If there are just two variables, the coefficients make up the r_{th} row of Pascal's Triangle (if the top is the 0_{th} row).

So, when there are two variables, the binomial coefficient can be used: $c_k = \binom{n}{r_{k,1}}$

$$A = \frac{(n+r-1)!}{r!(n-1)!}$$

The number of terms in the expansion is equal to n choose r without order, with repeats.

It is can be written more compactly

$$A = \left\langle {n \atop r} \right\rangle = \binom{n+r-1}{r}$$

as a multiset coefficient or binomial coefficient respectively.